The Non-Renewable Spaceship
Friday Forum

James A. Rising

Columbia SIPA

October 21, 2011

Overview

1. Introduction
2. Calculations
3. Next Steps

The Inspiration

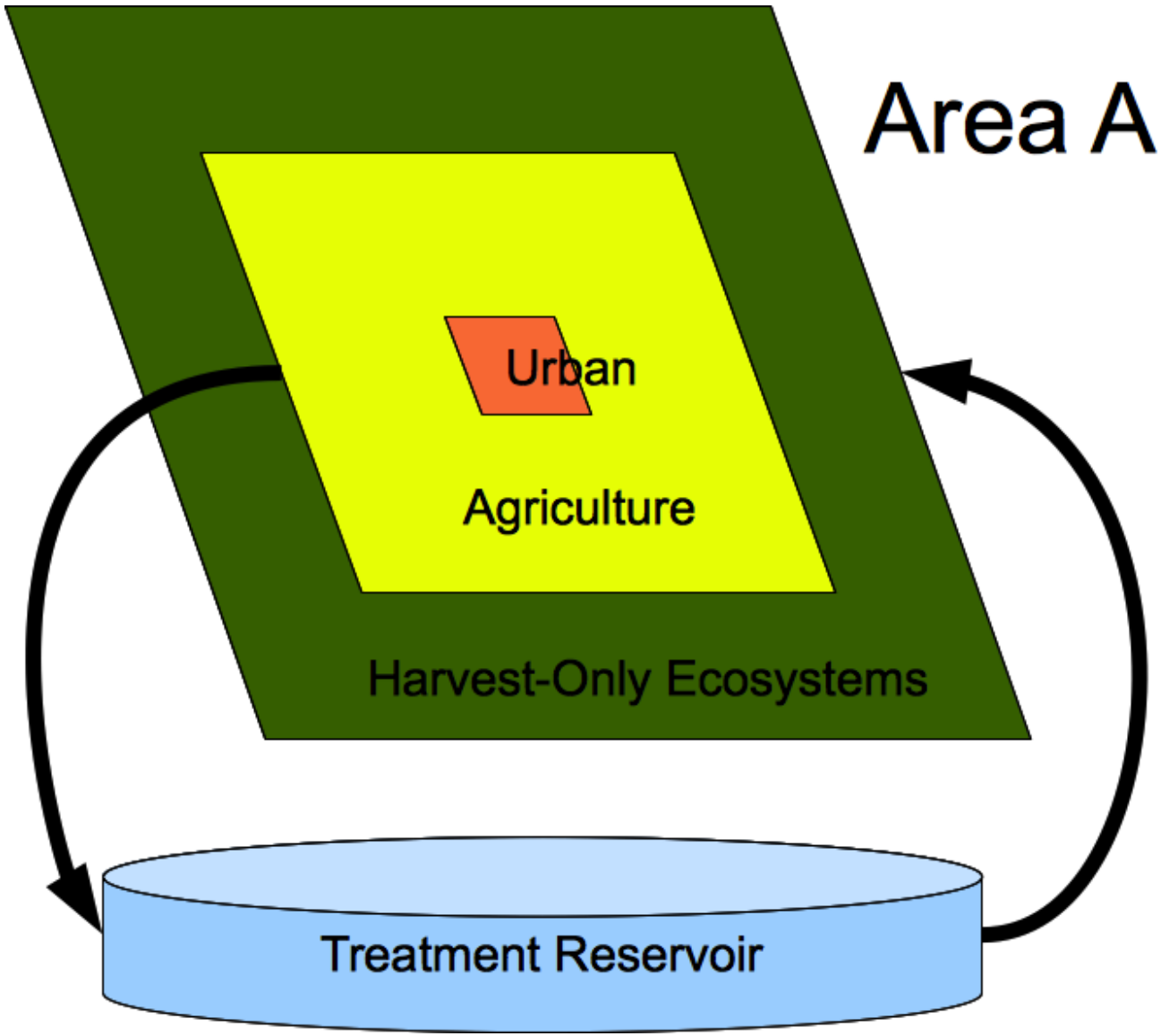
- Spaceship Earth
- What would you save?
- The need to set aside some non-renewables, in case weak sustainability doesn't suffice.

Framing the Problem

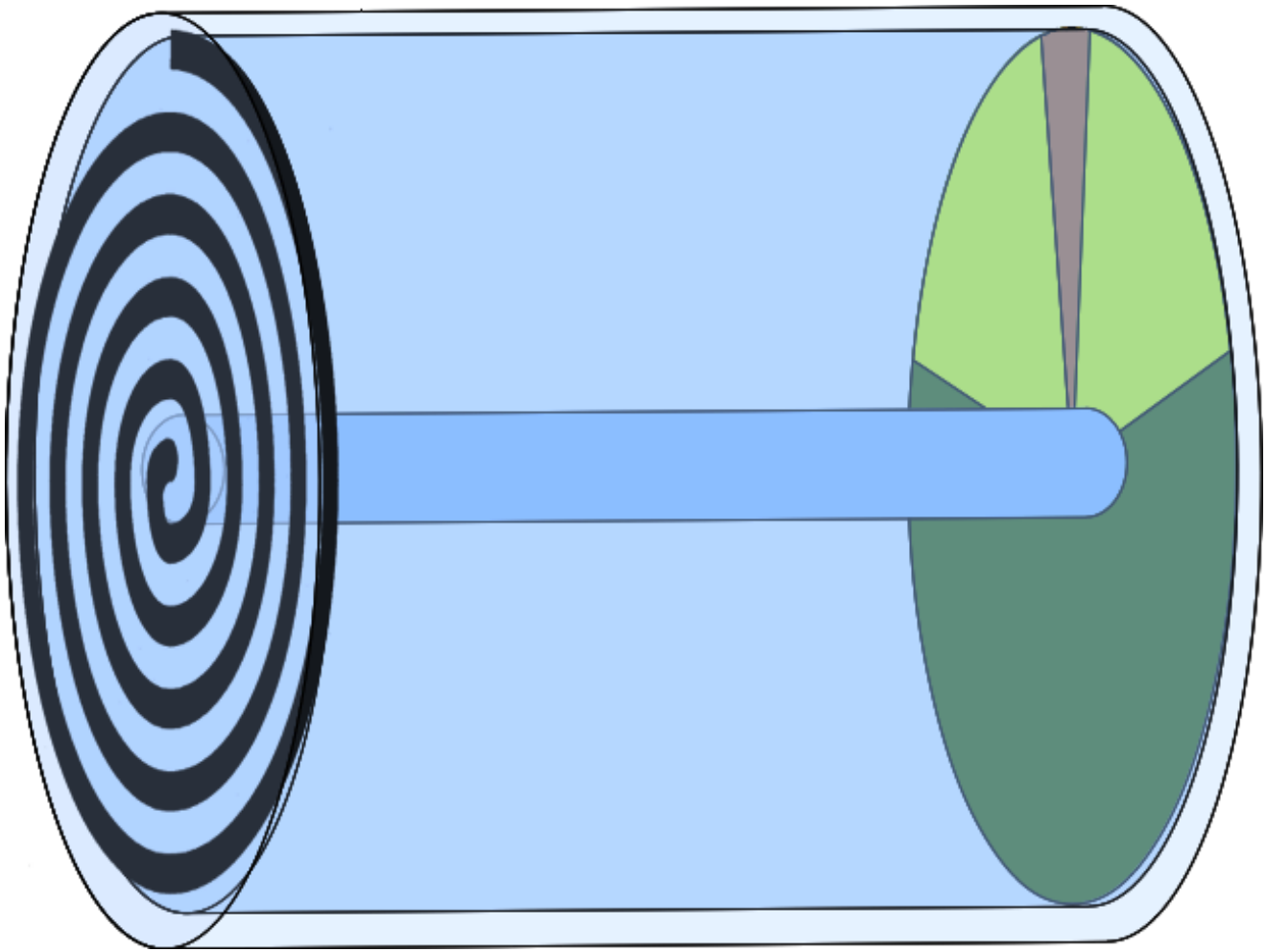
In case of a catastrophe, what is needed to save N people from Earth, and allow them to survive for a period of time, T, using only non-renewable resources?

- **Fossil fuels** for lift-off
- **Metal ores** for construction
- **Water and air** for people and ecosystem
- **Nuclear fuel** for energy requirements
- **Soil and Nutrients** for agriculture
- **Plants and Animals** for self-sustaining ecosystem

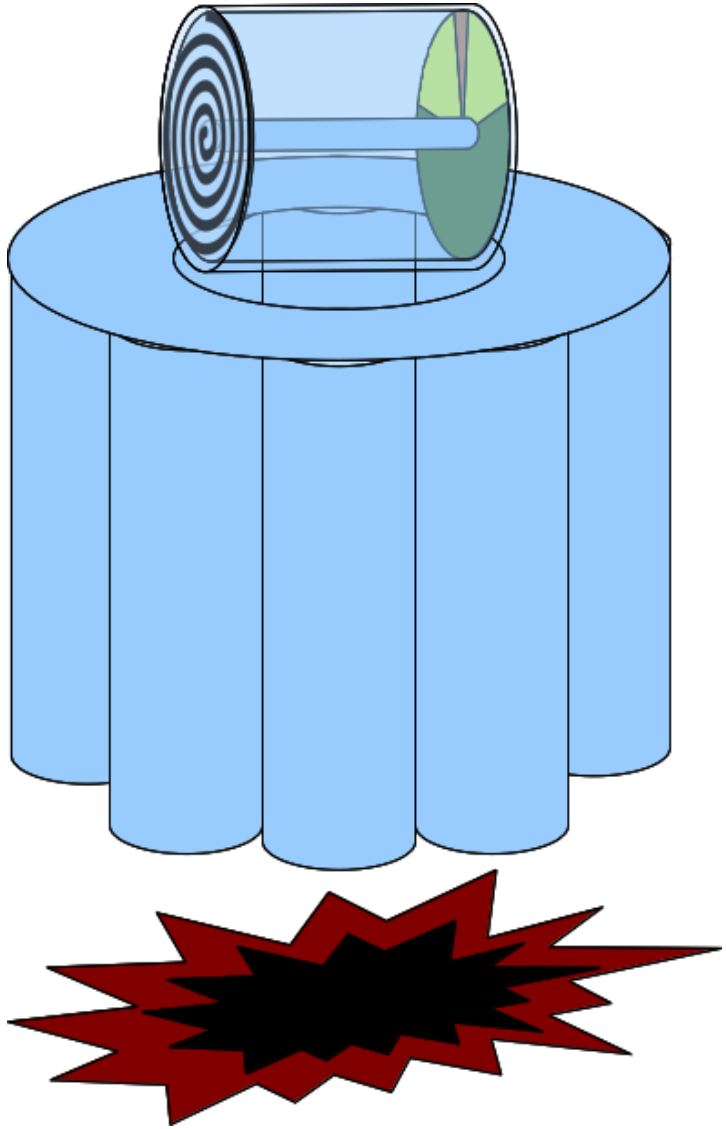
The Ship


Area A
Urban
Agriculture
Harvest-Only Ecosystems
Treatment Reservoir

The Ship



The Ship



Calculations

Total Area, Water Reservoir
$\rightarrow$ Ship Dimensions
$\rightarrow$ Nuclear Fuel
$\rightarrow$ Total Mass
$\rightarrow$ Fuels for Lift-off
Nature Area $\rightarrow$ Sustainable Species

`http://existencia.org/spaceship/index-old.html`

Total Area

Let $\gamma = \frac{A_{nature}}{A}$, the ratio of natural land area to total land area. On Earth, currently $\gamma \approx .6$ (or $1 - (1 - .6)(1 - .71) = .88$, including the oceans).

$$
\begin{aligned}
A_{ship} &= A_{urban} + A_{agriculture} + A_{nature} \\
A_{urban} &= NA_{home} \\
A_{agriculture} &= NA_{agpp} \\
A_{nature} &= \frac{\gamma}{1-\gamma} A_{agriculture}
\end{aligned}
$$

A_{home}	average urban area, per person	4 m^2	arbitrary
A_{agpp}	agricultural land, per person	500 m^2	see below

vegan diet	3000 ft^2	279 m^2	[?]
+ a few eggs/week	3500 ft^2	325 m^2	ibid
+ one chicken/week	24 300 ft^2	2 257 m^2	ibid
+ one cow/year	67 300 ft^2	6 252 m^2	ibid
current American dietary standards	1.2 acres	4 856 m^2	[?]

Water Reservoir

$$R_w = \frac{V_p + V_a}{365.25 \text{ days}} \implies V_w = NR_w T_w$$

V_p	water used by a person directly per year	5 m^3	[?]
V_a	water for agriculture per person per year	3000 m^3	[?]
T_w	time to process waste water	20 day	[?]
R_w	rate of water use by people	8.22 m^3/day	CALC
V_w	total volume of water onboard	1.645e5 m^3	CALC

Ship Dimensions

Constants: V (volume of core), A (area of spiral)
Variables: C (radius of core), R (radius of ship), L (length of spiral), W (width of ship), Θ (angular length of spiral)

$$A = WL$$
$$W = 2R$$
$$V = \pi C^2 W$$
$$r(\theta) = \frac{H\theta}{2\pi} + C$$
$$R = r(\Theta)$$
$$L = \int_0^{\Theta} r(theta) d\theta$$
$$\Longrightarrow L = \frac{A\left(\frac{V^3}{4AH\pi^2 + 4V\pi^2}\right)^{1/3}\pi}{V}$$

Nuclear Fuel

Consideration:

- Light for entire area, A_{ship}, at sun's intensity:

$$W_{light} = A_{ship} S \implies W_{electricity} = \frac{1}{\epsilon_{light}} A_{ship} S$$

- Heat needed to offset losses:

$$\frac{dT_{in}}{dt} = -\frac{1}{R} A_{outer} (T_{in} - T_{out}) + W_{heat} \implies E_{heat} = \frac{1}{R} A_{outer} \Delta T$$

$$m_{nuclear} = \frac{T}{J_{nuclear}} max(W_{electricity}, W_{heat})$$

ϵ_{light}	efficiency of fuel to light conversion	.3	arbitrary
S	light energy per m^2	2.5e3 W/m^2	sun
R	heat resistance coefficient $[Km^2/W]$	6	vacuum ins
$J_{nuclear}$	energy density of nuclear fuel	20e12 Ws/kg	Uranium 23

Total Mass

$$
\begin{aligned}
m_{ship} = & NM_{person} + A_{nature}(L_{water} D_{water} + C_{biomass}) + \\
& (A_{nature} + A_{agriculture}) L_{soil} D_{soil} + \\
& V_{water} D_{water} + \\
& A_{ship}(\max W_{electricity}, W_{heat}) \frac{T}{J_{nuclear}} + \\
& C_{inner} A_{ship}(1 + L_{level}(4L_{room})/(L_{room}^2)) + \\
& C_{outer}(\pi R_{ship}^2 + 2\pi R_{ship} W_{ship})
\end{aligned}
$$

M_{person}	average mass of a person and belongings	100 kg	arbitra
L_{water}	depth of water table	.5 m	arbitra
D_{water}	density of water	1000 kg/m^3	
$C_{biomass}$	Average of biomass per area	10 kg/m^2	see bel

tropical rain forest	20 kgC/m^2	45 - 111 kg/m^2	[?]
temperate deciduous forest	13.5 kgC/m^2	30.7 - 75 kg/m^2	ibid

Fuels for Lift-Off

Space Elevator Calculation, Potential Energy = Chemical Energy:

$$-\int_{\infty}^{R_e} \frac{GM_e m_{ship}}{r^2} dr = \frac{GM_e m_{ship}}{R_e}$$

$$\implies M_{fuel} = m_{ship} \frac{GM_e}{J_{fossil} R_e}$$

Fuels for Lift-Off

∞-Stage Thrusters, in terms of $m(r)$:

$$m(r) = \Delta m_f + m(r+\Delta)$$

$$\int_r^{r+\Delta} \frac{GM_e m(r)}{r^2} dr = -GM_e m(r)\left(\frac{1}{r+\Delta} - \frac{1}{r}\right) \approx \frac{GM_e m(r)\Delta}{r^2}$$

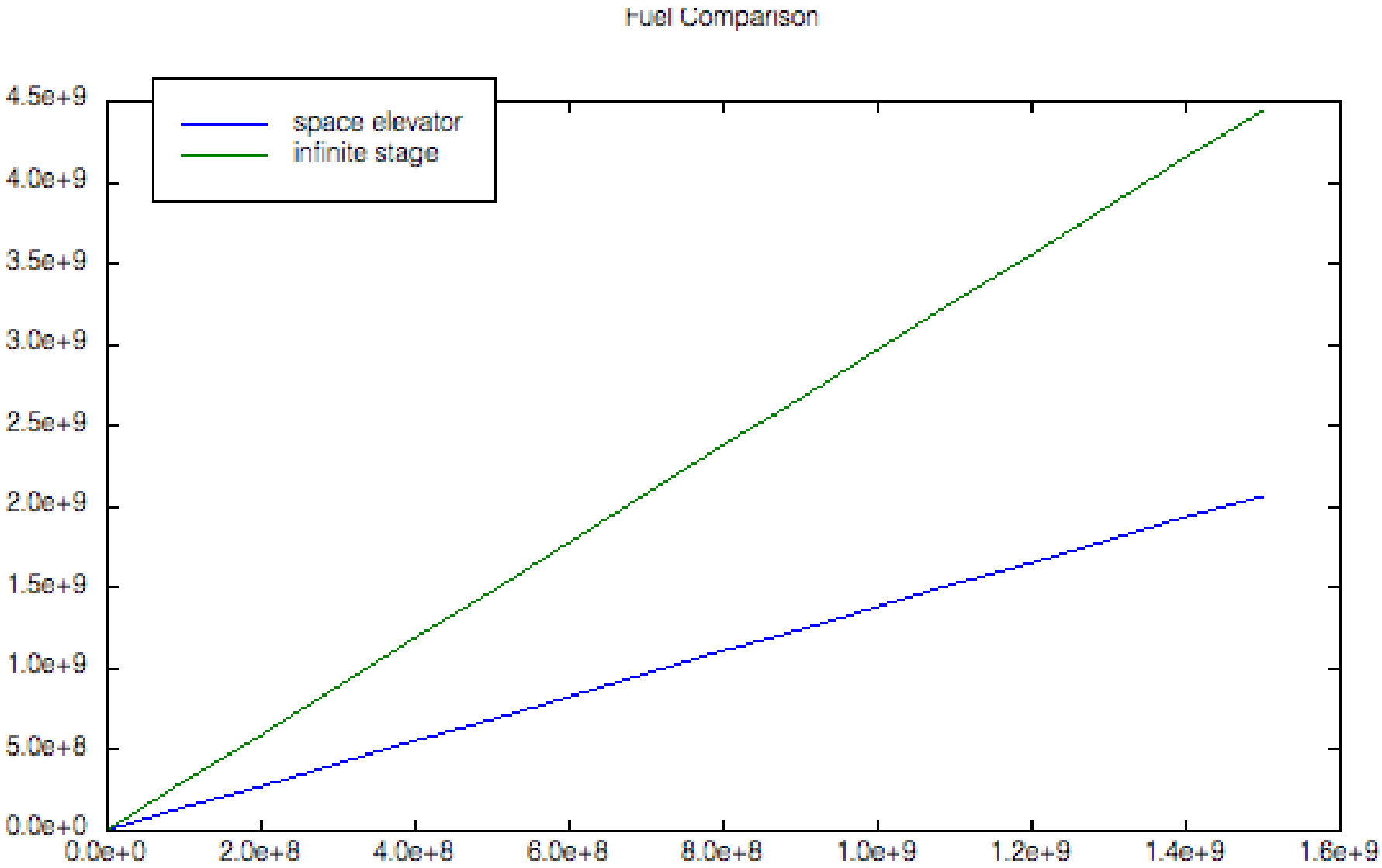
$$-J\Delta m = \frac{GM_e m(r)\Delta}{r^2} \implies \frac{dm}{dr} = -\frac{GM_e m(r)}{Jr^2}$$

$$\implies \frac{dm}{GM_e m(r)} = -\frac{dr}{Jr^2} \implies m(r) = Ce^{\frac{GM_e}{Jr}}$$

$$m(\infty) = C = m_{ship}$$

$$m_f = m(R_e) - m_{ship} = m_{ship}\left(e^{\frac{GM_e}{JR_e}} - 1\right)$$

Fuel Comparisons



Sustainable Species

From Darlington '57, bird species grow as $3.29A^{.301}$, with area in mi^2. For others, assume a constant ratio:

$$\begin{aligned} birds &= 3.29A^{.301} \\ mammals &= \frac{5490}{9998} birds \\ fish &= \frac{31300}{9998} birds \\ plants &= \frac{321212}{9998} birds \end{aligned}$$

Results

Population	1	10	100	1000	10000
Ship Area	.31 acres	3.1 acres	31 acres	310 acres	3099 acres
Ship Width	16 m	35 m	75 m	161 m	348 m
Spiral Rotations	2.5	5.4	11.6	25.1	54.0
Ship Mass	1.4e6 kg	1.38e7 kg	1.37e8 kg	1.36e9 kg	1.36e10 kg
Barrels Oil (x1000)	30	292	2898	28881	288354
World Consumption	3.5 hours	1.3 days	13 days	125 days	3.4 years
Water Volume	540 m^3	5395 m^3	53954 m^3	539545 m^3	5395448 m^3
Metal Material	103 tons	810 tons	7092 tons	66238 tons	640649 tons
Nuclear Fuel	1818 lbs	18176 lbs	181758 lbs	1817584 lbs	18175841 lbs
Bird Species	0	1	1	3	5
Plant Species	11	21	42	85	170

Costs

Population	1	10	100	1000	10000
Metal Meterials ($mil)	.088	.690	6.04	56.4	546
Nuclear Fuel ($mil)	.091	.909	9.09	90.9	909
Oil Cost ($mil)	2.97	29.2	290	2888	28835
Total Cost ($mil)	3	31	305	3035	30290
Cost per person ($mil)	3.14	3.08	3.05	3.04	3.03

To calculate

- Add mass of a nuclear power plant
- Calculate gravity and centripital forces

What's Missing

- "unrolled" spaceship on the website
- dynamic barrel piles, for different resources
- building costs and infrastructure
- maintenance workers and their equipment

Contributions